Street-Fighting Mathematics

The Art of Educated Guessing and Opportunistic Problem Solving

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Foreword by Carver A. Mead

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Preface

Too much mathematical rigor teaches *rigor mortis:* the fear of making an unjustified leap even when it lands on a correct result. Instead of paralysis, have courage—shoot first and ask questions later. Although unwise as public policy, it is a valuable problem-solving philosophy, and it is the theme of this book: how to guess answers without a proof or an exact calculation.

Educated guessing and opportunistic problem solving require a toolbox. A tool, to paraphrase George Polya, is a trick I use twice. This book builds, sharpens, and demonstrates tools useful across diverse fields of human knowledge. The diverse examples help separate the tool—the general principle—from the particular applications so that you can grasp and transfer the tool to problems of particular interest to you.

The examples used to teach the tools include guessing integrals without integrating, refuting a common argument in the media, extracting physical properties from nonlinear differential equations, estimating drag forces without solving the Navier–Stokes equations, finding the shortest path that bisects a triangle, guessing bond angles, and summing infinite series whose every term is unknown and transcendental.

This book complements works such as *How to Solve It* [37], *Mathematics and Plausible Reasoning* [35, 36], and *The Art and Craft of Problem Solving* [49]. They teach how to solve exactly stated problems exactly, whereas life often hands us partly defined problems needing only moderately accurate solutions. A calculation accurate only to a factor of 2 may show that a proposed bridge would never be built or a circuit could never work. The effort saved by not doing the precise analysis can be spent inventing promising new designs.

This book grew out of a short course of the same name that I taught for several years at MIT. The students varied widely in experience: from first-year undergraduates to graduate students ready for careers in research and teaching. The students also varied widely in specialization:

from physics, mathematics, and management to electrical engineering, computer science, and biology. Despite or because of the diversity, the students seemed to benefit from the set of tools and to enjoy the diversity of illustrations and applications. I wish the same for you.

How to use this book

Aristotle was tutor to the young Alexander of Macedon (later, Alexander the Great). As ancient royalty knew, a skilled and knowledgeable tutor is the most effective teacher [8]. A skilled tutor makes few statements and asks many questions, for she knows that questioning, wondering, and discussing promote long-lasting learning. Therefore, questions of two types are interspersed through the book.

Questions marked with a ▶ in the margin: These questions are what a tutor might ask you during a tutorial, and ask you to work out the next steps in an analysis. They are answered in the subsequent text, where you can check your solutions and my analysis.

Numbered problems: These problems, marked with a shaded background, are what a tutor might give you to take home after a tutorial. They ask you to practice the tool, to extend an example, to use several tools together, and even to resolve (apparent) paradoxes.

Try many questions of both types!

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Bon voyage

As our first tool, let's welcome a visitor from physics and engineering: the method of dimensional analysis.